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BNBUSMTT2/02/2023

**Total Marks: 60**

**Duration: Two [redacted] hours**

**N.B.** **All questions are compulsory.**

**Q.1 A)** **Attempt any One of the following:** **(8)**

(i) (a)State Euclid's lemma

(b) Let p is a prime number Suppose that $p \mid a_1.a_2 \cdot a_3 \dots a_n$

where $a_1., a_2, a_3 \dots , a_n \in \mathbb{Z}$ then $i$ with $1 \leq i \leq n$ such that $p \mid a_i$ equivalently If p | $a_1.a_2 \cdot a_3 \dots a_n$then $p \mid a_1 \; or \; p \mid a_2 \dots or \; p \mid a_n$

(ii) State and Prove Binomial theorem.

**B)** **Attempt any Two of the following:** **(8)**

(i) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$Prove that by induction Principal $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for any $n \in \mathbb{N}$.

(ii) Find GCD of 423 and 2763 also find $m$ and $n$ such that $d = m(423) + n(2763)$

(iii) Show that $55n + 2$ and $22n + 1$ are relatively primes $\forall \, n \in \mathbb{N}$

(iv) Find the last digit of $13^{207}$

**Q.2 A)** **Attempt any One of the following:** **(8)**

(i) Define Inverse of a function and if $f: X \to Y$ is a bijective function then Prove that $f$ is invertible.

(ii) Define equivalence relation
Let R be an equivalence relation on X then Prove that any two equivalence classes are either identical (same) or disjoint.

**B)** **Attempt any Two of the following:** **(8)**

(i) Let $f: \mathbb{R} \to \mathbb{R}, f(x) = 2x + 5 \; and \; g: \mathbb{R} \to \mathbb{R}, g(x) = \frac{x-5}{2}$ verify $g = f^{-1}$

(ii) Let $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x + 4$ and g: $\mathbb{R} \to \mathbb{R}$ given by $g(x) = x^2$ find $gof(1)$and $fof(1)$ .

(iii) In $X = \mathbb{Z}$ for $a, b \in \mathbb{Z}$ , $a * b = a + b - 3$ , Check whether * is commutative, associative and also find identity and inverse if they exist

(iv) In $\mathbb{Z}$ $aRb$ if and only if 5|a − b in $\mathbb{Z}$ find equivalence classes [0] and [1] under the relation R

Q.3 A) **Attempt any One of the following:** **(8)**

(i) State and prove Remainder theorem
Also find the remainder when $g(x) = x - 1$ is divided by
$f(x) = 16x^4 - 7x^3 + 5x + 1$ by using remainder theorem.

(ii) (a) State Euclidean algorithm for polynomials.
(b) let $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \in \mathbb{Z}[x]$ and $a_n \neq 0$ prove that
$\frac{p}{q} \in \mathbb{Q}$ is a root of $f(x)$ then $p\,|a_0$ and q $|a_n$.

B) **Attempt any Two of the following:** **(8)**

(i) Find the GCD of the polynomial $(x) = x^8 - 1$ and $g(x) = x^6 - 1$

(ii) Find quotient and remainder when $f(x) = x^5 - 3x^4 - 2x^2 + 4x + 2$ and
$g(x) = x^2 + 2x + 2$

(iii) Find all roots of $f(x) = x^3 - 2x^2 - 4x + 8$ if $r_1 + r_2 = 0$

(iv) State factor theorem and Using factor theorem check whether
$f(x) = x - 2$, is a factor of $g(x) = 6x^3 - 7x + 1$

Q.4 **Attempt any Three of the following:** **(12)**

(i) For integers $a, b, c$ prove that If 9 | (b + c) and 9 | (b − c) then
9 | (50 b − 22 c).

(ii) Find the least positive integer to which $2^{758}$ is congruent modulo 63

(iii) Prepare the Multiplication table for $\mathbb{Z}_6$

(iv) Check whether the function $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 10x + 7$ is bijective

(v) Find 4th roots of unity.

(vi) Find the multiplicity of each root of the polynomial
$f(x) = x^4 - 5x^3 + 9x^2 - 7x + 2$

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