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F. Y. B. Sc. SEMESTER II MATHEMATICS PAPER II

UNIT 1: EXTRA PROBLEMS

SECTION I: Multiple Objective questions:

1. Equation of a line is
   (a) $ax + by = c$ (c) $y = 4ax$
   (b) $x^2 + y^2 = r^2$ (d) $ax + by + cz = d$

2. Equation of a plane is
   (a) $ax + by = c$ (c) $y = 4ax$
   (b) $x^2 + y^2 = r^2$ (d) $ax + by + cz = d$

3. Consider the system $2x - y + 3z = 0$
   $x + 2y - z = 0$
   (a) m=2, n=2 (c) m=2, n=3
   (b) m=3, n=3 (d) m=3, n=2

4. If A is a matrix of order $m \times n$, then order of its transpose A^T is
   (a) $m \times m$ (c) $n \times n$
   (b) $n \times m$ (d) $m \times n$

5. Which of the following is not a trivial solution?
   (a) (0,0) (c) (0,0,0)
   (b) (0,0,…,0) (d) (1,0,…,0)

6. Which of the following is not a diagonal matrix?
   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$
   (b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (d) None of the above.

7. Order of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ is
   (a) 6 (c) 2×3
   (b) 3×3 (d) 3×2

8. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -1 & 5 \end{bmatrix}$ then
   (a) AB exists (c) BA exists
   (b) $A + B$ exists (d) $A - B$ exists

9. Which of the following is true for matrices A, B and C?
   (a) $A + B = B + A$ (c) $(AB)C = A(BC)$
   (b) $(A + B) + C = A + (B + C)$ (d) All of the above

10. Which of the following is false for matrices A, B and C?
    (a) $A + B = B + A$ (c) $AB = BA$

(b) $(A+B)+C=A+(B+C)$ (d) $(AB)C=A(BC)$

11. Solution of the system $\begin{matrix}3x+2y=0\\5x-y=0\end{matrix}$ is
(a) (0,0) (c) (1,1)
(b) (2,-3) (d) None of the above.

12. Which of the following is an elementary row operation?
(a) R_{ij} (c) αR_i
(b) $R_i+\alpha R_j$ (d) None of the above.

13. Which of the following is diagonal matrix as well as scalar matrix?
(a) $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ (c) $\begin{pmatrix}4&0\\0&0\end{pmatrix}$
(b) $\begin{pmatrix}1&0\\0&2\end{pmatrix}$ (d) $\begin{pmatrix}0&1\\1&0\end{pmatrix}$

14. Property: $A+B=B+A$ is called
(a) Commutative law (c) Associative law
(b) Distributive law (d) None of the above.

15. Order of the matrix $\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}$ is
(a) 6 (c) 2×2
(b) 3×3 (d) 2×3

16. Property: $A(B+C)=AB+AC$ is called
(a) Commutative law (c) Associative law
(b) Distributive law (d) None of the above.

17. Which of the following is false for matrices A and B?
(a) $(A^T)^T=A$ (c) $(A-B)^T=A^T-B^T$
(b) $(A+B)^T=A^T+B^T$ (d) None of the above.

18. Which of the following is not an upper triangular matrix?
(a) $\begin{pmatrix}9&2\\0&3\end{pmatrix}$ (c) $\begin{pmatrix}9&2\\0&0\end{pmatrix}$
(b) $\begin{pmatrix}9&0\\0&3\end{pmatrix}$ (d) $\begin{pmatrix}9&0\\2&0\end{pmatrix}$

19. If A and B are symmetric matrices, which of the following matrix is not symmetric?
(a) $A+B$ (c) αB
(b) αA (d) AB

20. Which of the following is in Row echelon form?
(a) $\begin{pmatrix}1&4\\0&1\end{pmatrix}$ (c) $\begin{pmatrix}1&2\\0&0\end{pmatrix}$
(b) $\begin{pmatrix}1&0&0\\0&0&1\end{pmatrix}$ (d) All of the above

21. For the system with the Augmented matrix $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, it has

(a) Infintely many solutions
(b) Unique solution
(c) No solution
(d) Non-trivial solution

22. Which of the following is not a lower triangular matrix?

(a) $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

23. If A is invertible, which of the following is false?

(a) $\det A = 0$
(b) $(A^T)^{-1} = (A^{-1})^T$
(c) $(A^{-1})^{-1} = A$
(d) $AA^{-1} = I = A^{-1}A$

24. Which of the following is a Symmetric matrix?

(a) $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$
(b) $\begin{pmatrix} 3 & 3 \\ -4 & -4 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$
(d) $\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$

25. Consider the system $\begin{aligned} 2x - y &= 9 \\ x + 2y &= -1 \\ 9x - y &= 0 \end{aligned}$

(a) m=2, n=2
(b) m=3, n=3
(c) m=2, n=3
(d) m=3, n=2

26. For a system of m linear equations in n unknowns, the order of the augmented matrix is

(a) $m \times n$
(b) $n \times m$
(c) $m \times (n+1)$
(d) $(m+1) \times n$

27. Augmented matrix of the system $\begin{aligned} x + y - 2z &= 0 \\ -x + 4y + z &= 0 \end{aligned}$ is:

(a) $\begin{pmatrix} 1 & 1 & -2 & 9 \\ -1 & 4 & 1 & 0 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 4 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & -1 \\ 1 & 4 \\ -2 & 1 \\ 9 & 0 \end{pmatrix}$
(d) None of the above.

28. Which of the following is in Row echelon form?

(a) $\begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

29. Which of the following is not a Symmetric matrix?
(a) $\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ (c) $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$
(b) $\begin{pmatrix}-2 & 3\\ 3 & -6\end{pmatrix}$ (d) $\begin{pmatrix}1 & 2\\ 1 & 2\end{pmatrix}$

30. Which of the following is not a diagonal matrix?
(a) $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ (c) $\begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix}$
(b) $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$ (d) $\begin{pmatrix}0 & 1\\ 2 & 0\end{pmatrix}$

31. Which of the following is not in Row echelon form?
(a) $\begin{pmatrix}1 & 1 & -2\\ 0 & 0 & 0\end{pmatrix}$ (c) $\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$
(b) $\begin{pmatrix}1 & 2 & 3\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$ (d) $\begin{pmatrix}2 & 3 & 4\\ 0 & 5 & 6\\ 0 & 0 & 7\end{pmatrix}$

32. Which of the following is not a Skew symmetric matrix?
(a) $\begin{pmatrix}1 & -1\\ 2 & -2\end{pmatrix}$ (c) $\begin{pmatrix}0 & 1\\ 2 & 0\end{pmatrix}$
(b) $\begin{pmatrix}0 & -4\\ -4 & 0\end{pmatrix}$ (d) $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

33. If $A = A^T$, then matrix A is called
(a) Symmetric matrix (c) Transpose of a matrix
(b) Skew-symmmetric matrix (d) None of the above.

34. For the system with the Augmented matrix $\left(\begin{array}{ccc|c}1 & -8 & 7 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{array}\right)$, it has
(a) Infintely many solutions (c) No solution
(b) Unique solution (d) Non-trivial solution

35. Which of the following is not an upper triangular matrix?
(a) $\begin{pmatrix}1 & 3\\ 0 & 2\end{pmatrix}$ (c) $\begin{pmatrix}0 & 0 & 2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$
(b) $\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}$ (d) $\begin{pmatrix}1 & 9 & 2\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}$

36. Property: $(A + B) + C = A + (B + C)$ is called
(a) Commutative law (c) Associative law
(b) Distributive law (d) None of the above.

37. If $A = -A^T$, then matrix A is called
(a) Symmetric matrix (c) Transpose of a matrix

(b) Skew-symmmetric matrix (d) None of the above.

38. Matrix obtained by interchange of rows and columns is called
(a) Transpose (c) Symmetric
(b) Inverse (d) Skew-symmetric

39. Which of the following is an elementary row operation?
(a) $R_i + \alpha R_j$ (c) R_{ij}
(b) αR_i (d) None of the above

40. Which of the following is in Row echelon form?
(a) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

41. For the system with the Augmented matrix $\left(\begin{array}{ccc|c} 1 & -8 & 7 & 9 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$, it has
(a) Infintely many solutions (c) No solution
(b) Unique solution (d) Non-trivial solution

42. Which of the following matrix is both symmetric matrix and skew-symmetric matrix?
(a) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

43. Which of the following is not in Row echelon form?
(a) $\begin{pmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

44. For the system with the Augmented matrix $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right)$, it has
(a) Infintely many solutions (c) No solution
(b) Unique solution (d) Non-trivial solution

45. System is called inconsistent if it has
(a) Infintely many solutions (c) No solution
(b) Unique solution (d) Non-trivial solution

46. For the system with the Augmented matrix $\left(\begin{array}{ccc|c} 1 & -1 & 7 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -8 \end{array}\right)$, it has

(a) Infintely many solutions
(b) Unique solution
(c) No solution
(d) Non-trivial solution

47. Which of the following matrix is both diagonal matrix and scalar matrix?

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$
(d) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

48. Which of the following is in Row echelon form?

(a) $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$
(d) All of the above

49. Row echelon form of $\begin{pmatrix} 2 & 4 & 6 \\ -1 & -2 & -3 \end{pmatrix}$ is

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$
(d) None of the above

50. For the system with the Augmented matrix $\left(\begin{array}{ccc|c} 1 & -2 & 5 & -1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right)$, it has

(a) Infintely many solutions
(b) Unique solution
(c) No solution
(d) Non-trivial solution

51. Which of the following matrix is both lower triangular and upper triangular?

(a) $\begin{pmatrix} 1 & -2 & 3 \\ 0 & 8 & 9 \\ 0 & 0 & -4 \end{pmatrix}$
(b) $\begin{pmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 0 & 0 \\ 8 & -1 & 0 \\ -7 & 1 & 2 \end{pmatrix}$
(d) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

52. Which of the following is not in Row echelon form?

(a) $\begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
(d) All of the above

53. For the system with the Augmented matrix $\left(\begin{array}{ccc|c} 1 & -8 & 7 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$, it has

(a) Infintely many solutions
(b) Unique solution
(c) No solution
(d) Non-trivial solution

54. Which of the following is not in Row echelon form?

(a) $\begin{pmatrix} 1 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$
(b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

55. Which of the following is false for matrices A and B?

(a) $(A^T)^T = A$
(b) $(A+B)^T = A^T + B^T$
(c) $(A-B)^T = A^T - B^T$
(d) $(AB)^T = A^T B^T$

Problems:

Q.1 Solve the following homogeneous system using elimination method:

1. $\begin{aligned} x - y + z &= 0 \\ x + 2y + z &= 0 \\ 2x + y + 3z &= 0 \end{aligned}$
2. $\begin{aligned} 2x - y + 3z &= 0 \\ 3x + 2y + z &= 0 \\ x - 4y + 5z &= 0 \end{aligned}$
3. $\begin{aligned} (1-k)x - y + z &= 0 \\ 2x + (1-k)y &= 0 \\ 2y - (1+k)z &= 0 \end{aligned}$
4. $\begin{aligned} x + ky + z &= 0 \\ 3x + ky + z &= 0 \\ kx + 4y + kz &= 0 \end{aligned}$

Q.2 Reduce to row echelon form:

1. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{pmatrix}$
2. $\begin{pmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 10 & 8 & 6 \\ 2 & 0 & -2 \\ -6 & -8 & -10 \end{pmatrix}$
4. $\begin{pmatrix} 0 & 2 & 1 \\ 3 & 5 & 1 \\ 5 & -1 & 2 \\ 2 & 6 & 5 \end{pmatrix}$

Q.3 Solve the following systems using Gauss-elimination method:

1. $\begin{aligned} x + y + z &= 3 \\ x + y - z &= 1 \\ x + y + 7z &= 6 \end{aligned}$
2. $\begin{aligned} x + y + z &= 3 \\ x + y - z &= 1 \\ x + y + 4z &= 6 \\ x + y - 4z &= -1 \end{aligned}$
3. $\begin{aligned} 4x - 5y - 2z &= 2 \\ 5x - 4y + 2z &= -2 \end{aligned}$
4. $\begin{aligned} 5x + 3y + 7z - 4 &= 0 \\ 3x + 26y + 2z - 9 &= 0 \end{aligned}$

$2x + 2y + 8z = 1$ $7x + 2y + 10z - 5 = 0$