

SYBA Sem IV

(3 Hours)

Marks : 100

25

27/4/19

N.B. 1. All questions are compulsory.

2. Attempt any two sub-questions out of three sub questions from Q.2, Q.3, Q.4, Q.5.

3. Figures to the right indicate full marks.

4. Use of non-programmable scientific calculator is allowed.

5. Standard Normal probability table values are provided at the end of the question paper.

2-30-5-30

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Q.1. a. Answer ANY FIVE from the following:

(10)

- If  $f(x)$  is a p.d.f of a continuous random variable then  $f(x)$  may be  $\geq 1$ . Is the statement true? Correct if necessary.
- If  $X$  has a p.d.f  $f(x) = kx^3$  for  $0 \leq X \leq 1$ , zero elsewhere, then the value of  $k$  is 3. Is the statement true? Correct if necessary.
- If  $X \sim N(\mu, \sigma^2)$  then  $P(X \leq a) = P(X \geq b)$ . Is the statement true? Correct if necessary.
- If  $X \sim U(4, 12)$ , then what is the value of mean of  $X$ ?
- Critical region is a region of rejection. Is the statement true? Correct if necessary.
- $H_1: \mu > 10$  is two tailed test. Is the statement true? Correct if necessary.
- Power of the test is size of type 1 error.

b. Answer ANY FIVE from the following in one/two sentences:

(10)

- If  $X$  is a continuous random variable then what is the value of  $P(X = x)$ ?
- What is the median of the distribution in terms of c.d.f?
- If  $X \sim U(-4, 4)$  then what is the value of  $F(0)$ ?
- If  $Z \sim N(0, 1)$  then write the p.d.f of  $Z$ .
- Define unbiased estimator.
- State the two types of estimation.
- Define type 1 error

Q.2. Attempt ANY TWO sub-questions:

(20)

a)

- Define raw moments and central moments of a continuous random variable  $X$ . Also state the relationship between the 3<sup>rd</sup> central moment with raw moments. (05)
- A spinner has a density function

$$f(x) = \frac{1}{360} \quad 0 \leq X \leq 360$$

$$= 0 \quad \text{otherwise.}$$

Find c.d.f and hence find  $P(90 < X < 180)$  using both functions  $f(x)$  and  $F(x)$ . (05)

b)

- State the formulae for skewness and Kurtosis in terms of moments for a continuous random variable  $X$ . (03)
- Let  $x$  be a continuous random variable with p.d.f given by (07)

$$f(x) = k(x - 1)^3 \quad \text{for } 1 < x < 3$$

$$= 0, \quad \text{otherwise.}$$

Find

- K
- Mean
- Variance

c) A continuous random variable X has the following p.d.f.

$$f(x) = \begin{cases} \frac{2x}{3} & 0 \leq x < 1 \\ \frac{2(2-x)}{3} & 1 \leq x < 2 \\ \frac{2x(3-x)}{7} & 2 \leq x < 3 \\ 0, & \text{otherwise.} \end{cases}$$

Find Median and first quartile.

(10)

Q.3. Attempt ANY TWO sub-questions:

(20)

a) Define exponential distribution with parameter  $\lambda$ . Find mean and variance.

(10)

b)

i. State six important properties of normal distribution.

(06)

ii. If  $X \sim U(-a, a)$ ,  $a > 0$ , find a such that

$$P(X \leq 0.5) = 0.7. \text{ Also find } P(0.3 < X < 0.7)$$

(04)

c)

i. Write a short note on normal approximation to Binomial distribution.

(03)

ii. In a particular faculty 60% of the students are men and 40% are women. In a random sample of 50 students Using Normal distribution find what is the probability that

- More than half are women
- Less than 18 were women

(07)

Q.4. Attempt ANY TWO sub-questions:

(20)

a) Define the following terms with suitable example.

i. Null hypothesis

ii. Composite hypothesis

iii. Level of significance

iv. Simple hypothesis

v. Two tailed test

(10)

b)

i. Explain sampling distribution of sample proportion.

(05)

ii. Suppose we would like to determine if the typical amount spent per customer for dinner at a new restaurant in town is more than Rs. 50. A sample of 49 customers over a three week period was randomly selected and the average amount spent was Rs. 62. Assume that the standard deviation is known to be Rs. 10. Using a 0.02 level of significance, would we conclude the typical amount spent per customer is more than Rs.50?

(05)

c)

i. Explain how would you arrive at a best decision criterion based on a large sample to test the hypothesis  $H_0: P = P_0$  against an alternative  $H_1: P > P_0$

(05)

ii. Let random variable X has binomial distribution with  $n = 10$  and  $P$ , where  $P$  can be either  $\frac{1}{2}$  or  $\frac{1}{4}$ . We select a random sample of size 1, and if the observed value is less than or equal to 3, we reject that  $P = \frac{1}{2}$  and accept that  $P = \frac{1}{4}$ . Find level of significance and power of the test.

(05)

Q.5. Attempt ANY TWO sub-questions: (20)

a) i. Define cumulative distribution function of a random variable and state its properties. (05)

ii. The density function of X is  
 $f(x) = a + bx^2 \quad 0 \leq x \leq 1$   
 $= 0, \quad \text{otherwise.}$   
 If  $E(x) = 3/5$ , find a and b. (05)

b) i. Define uniform random variable over an interval (a,b). Find its distribution function. (04)

ii. The time it takes an international telephone operator to place an overseas call is normally distributed with mean 43 seconds and standard deviation 8 seconds. What is the probability that

- my call will go through in less than 1 minute.
- I will take more than 35 seconds to place a call. (06)

c) i. Derive  $100(1-\alpha)\%$  confidence interval for population mean  $\mu$ . (05)

ii. A study of noise levels in hospitals produced the following data. Test the claim that the noise level in the corridors is higher than in the clinics. Use  $\alpha = 0.05$ . (05)

Sample	Corridors	Clinics
Size	84	34
Mean(db)	61.2	59.4
S.D	7.9	7.9

Table below gives area to the left of positive value of Z (standard normal variate)

Standard Continuous Distributions

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990