ws Oct. 2013-39
Con. 371–13. KN–1877

(3 Hours) [Total Marks : 100

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicates **full** marks.
(3) Draw neat **diagrams** wherever **necessary**.
(4) Symbols have their usual meanings unless otherwise stated.
(5) Use of **log table** or **non programmable** calculator is **allowed**.

1. Attempt any **two** :–
   (a) Describe the method to solve the second order homogeneous differential equation with constant coefficients. Hence solve $y'' + y' - 2y = 0$. **10**
   (b) State Fourier theorem and explain Fourier cosine and sine series. Expand $f(x) = x$ for $-\pi \leq x \leq \pi$ in Fourier series. Show the graphical representation of the function. **10**
   (c) State cosine and sine transform pairs. Find cosine and sine transform of $f(x) = e^{-x}$. **10**

2. Attempt any **two** :–
   (a) What are fermions ? Derive Fermi-Dirac distribution law. **10**
   (b) Consider two interactive systems A and A′. They are neither adiabatically isolated nor their external parameters are kept fixed. Show that the equilibrium temperature and pressure on both sides of the partition become equal. **10**
   (c) Use the concept of entropy to describe Gibbs free energy of a system. Hence explain co-existence of two phases, the phase equilibrium curve and the triple point. **10**

3. Attempt any **two** :–
   (a) Define central force. Derive the equation of motion of a particle of mass m under the action of a central force. Show that its angular momentum is conserved. **10**
   (b) A frame of reference S rotates with variable angular velocity $\bar{\omega}$ with respect to an inertial frame S′ fixed in space. Both are having common origin. **10**

   Show that $\left(\frac{d\bar{r}}{dt}\right)_{fix(s')} = \left(\frac{d\bar{r}}{dt}\right)_{rot(s)} + \bar{\omega} \times \bar{r}$ and

$$\left(\frac{d^2\bar{r}}{dt^2}\right)_{fix(s')} = \left(\frac{d^2\bar{r}}{dt^2}\right)_{rot(s)} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) + 2\bar{\omega} \times \left(\frac{d\bar{r}}{dt}\right)_{rot(s)} + \frac{d\bar{\omega}}{dt} \times \bar{r}$$

   (c) Derive Euler's equation of motion for an ideal fluid. **10**

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4. Attempt any **two** :–

   (a) Derive Lagrange's equation of motion for n variables $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$. **10**

   (b) What are Euler angles ? Explain the order in which the transformation of axes is carried out. Show that the kinetic energy of a symmetric top in terms of Euler angles can be written as – **10**

   $$T = \frac{1}{2} I\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) + \frac{1}{2} I_3 \left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2.$$

   (c) An anharmonic oscillator of mass m is subjected to a conservative force, $F(x) = -K(x + \alpha x^3)$. Illustrate graphically the potential energies of the 'hard' spring and 'soft' springs using $K = \pm 1$ and $\alpha = \pm 1$. Obtain reduced Duffing equation of motion. **10**

5. (a) Attempt any **two** :–

   (i) Describe the change of interval of the Fourier expansion. 5

   (ii) If the mean velocity of hydrogen molecule, at a given temperature is 1450 m/sec. Calculate their root mean square velocity and most probable velocity at the same temperature. 5

   (iii) Eight particles are randomly distributed in two boxes with equal probability. 5
   Find : (1) Total number of microstates and macrostates.
   (2) Which is most probable distribution ? Why ?
   (3) Probability of distribution (5, 3).
   (4) Number of microstates in macrostate (6, 2).

   (b) Attempt any **two** :–

   (i) For a particle moving under the action of a central force, the effective potential is given by $U_1 = \frac{-200}{r} + \frac{100}{r^2}$ (MKS units). Sketch roughly u_1 as a function of r and find the radius of the circular motion. 5

   (ii) Use Lagrangian formalism to find the differential equation for a compound pendulum, which oscillates in a vertical plane about a fixed horizontal axis. 5

   (iii) Show that in Logistic map for $\lambda = 0{\cdot}8$ the fixed point $x = 0$ is an attractor. Plot a suitable graph. 5