B.N.Bandodkar College of Science,Thane

F.Y.B.Sc First Semester End Examination Oct. 2011.

USMT 102

Duration:2hours **Max.Marks:60**

Instructions to the candidates:

1.All questions are compulsory.

2.Figures to the right indicate full marks.

Q.1 (a) Write down the expansion of $(a-b)^n$,for $n \geq 0$ and hence expand $(a-b)^6$. (3)

(b) Attempt any three of the following

(i) Prove that the number of primes is infinite. (4)

(ii) Find the greatest common divisor(g.c.d) of 497 and 3015.Express it in the form of 497x + 3015y, for suitable x and y. (4)

(iii) Prove that , for all $n \in$ □ , 21n + 4 and 14n + 3 are relatively primes. (4)

(iv) Prove that for $n \geq 4$, $2^n < n!$. (4)

(v) If a,b,c are integers such that $a \mid (b+c)$ and $a \mid (b-c)$,then prove that $a \mid (12b - 18c)$. (4)

Q.2 (a) Define cartesian product of two sets.Find Cartesian product AxB for A={a,b} and B={1,2,3}. (3)

(b) Attempt any three of the following

(i) Show that f: □ $\rightarrow$ □ given by f(x) = 2x+ 5 is a bijection and hence find its inverse. (4)

(ii) If X is a finite set and $f: X \rightarrow X$ is a function such that $f^2(x) = x$, for $\forall \; x \in X$ Show that f is a bijection. (4)

(iii) If 5 points are chosen in an equilateral triangle of side 2.Show that at least two points are at a distance of atmost one unit from each other. (4)

(iv) If A is a finite set having n elements then show that the total number of subsets of A is 2^n . (4)

(v) Define composite function.If $f(x) = x^2 + 5$ and $g(x) = 3x + 2$.Find composite functions fog and gof. (4)

Q.3 (a) Define congruence relation modulo n , with an example. (3)

P.T.O.

(b) Attempt any three of the following

(i) For any integer x, show that , $x^2 \equiv 0$ or 1 (mod 4). (4)

(ii) Prove that the relation of ' congruent modulo n' for any fixed positive integer n, is an equivalence relation in □ . (4)

(iii) Find the last digit of 7^{313} . (4)

(iv) Define Euler's ϕ function.Find $\phi(30)$ and ϕ (17). (4)

(v) Show that every four digit integer of the type abba, is divisible by 11. (4)

Q.4 (a) Prove by induction that there is no integer between 0 and 1. (3)

(b) Attempt any three of the following

(i) If (a,b) = 1 , $a \mid bc$ then prove that $a \mid c$. (4)

(ii) Express 1350 and 1176 as the product of the same prime powers and hence find their least common multiple(l.c.m.). (4)

(iii) On □ , define a binary operation '*' as a*b = 5a-3b, $\forall$ a,b in □ .Check whether * is commutative and associative. (4)

(iv) State addition and multiplication principles of counting. (4)

(v) Prove that, a positive integer is divisible by 9 ,if sum of its digits is divisible by 9. (4)

****