B. N. Bandodkar College of Science, Thane

Second Semester End Examination March 2015(Old Course)

Mathematics Paper (USMT 202)

Duration: 2 ½ hours Max. Marks: 75

N.B. 1) All questions are compulsory.

2) Figures to the right indicate full marks.

Q.1 (a) Prove that IN_n is not equivalent to a proper subset of itself. (8)

or

(a) Define equivalence of two sets. Give an example. Prove that the relation 'A equivalent to B' is an equivalent relation on a family of sets X. (8)

(b) Attempt any three of the following (12)

(i) Prove that the relation of ' congruent modulo n ' for any fixed positive integer n, is an equivalence relation on Z. (4)

(ii) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, $n \geq 3$ with $a_1 = 11$ and $a_2 = 31$. (4)

(iii) Define partition of a set. Explain with an example. (4)

(iv) Prove that any two equivalence classes of X are either identical or disjoint . (4)

Q.2 (a) State and prove the principle of inclusion and exclusion. (8)

or

(a) Find the number of r – digit sequences formed by using 1, 2, 3 and 4 in which each of the three digits 1,2 and 3 appears at least once. (8)

(b) Attempt any three of the following (12)

(i) State whether the permutation $\Pi = (1\ 2\ 3)(3\ 4\ 5)(1\ 2)(1\ 3\ 4\ 5)(3\ 5\ 1\ 2)$ is even or odd permutation in S_5. (4)

(ii) Define multinomial number. Calculate the coefficient of $x^4y^5z^6$ in the expansion of $(x + y + z)^{15}$. (4)

(iii) Find the total number of derangements on 5 symbols. (4)

(iv) How many different arrangements can be made by using all the letters of the word MATHEMATICS . (4)

Q.3 (a) Define associates and units in IR. If f(x) and g(x) are associates in IR[x], prove that f(x) = c. g(x), where c is a suitable constant in IR. (8)

or

(b) State and prove unique factorization theorem in IR[x] (8)

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(b)	**Attempt any three of the following**	(12)
(i)	Find the greatest common divisor (g.c.d) of $f(x) = x^2 - 1$ and $g(x) = x^3 + 2x^2 - x - 2$.	(4)
(ii)	Prove that the only unit polynomials in IR[x] are the non zero constant polynomials.	(4)
(iii)	Find the multiplicity of each root of $f(x) = x^4 - x^3 - 3x^2 + 5x - 2$.	(4)
(iv)	State and prove division algorithm theorem in IR[x].	(4)

(b)	**Attempt any three of the following**	(15)
(i)	Let X be a finite set and $f: X \rightarrow X$ is given by $f^2(x) = x$, for $x \in X$. Show that f is a bijection.	(5)
(ii)	Show that $x^2 - 5$ is reducible in IR[x].	(5)
(iii)	State and prove rational root theorem.	(5)
(iv)	Find the fifth roots of unity.	(5)
(v)	Find the inverse of $\sigma = (5\ 3\ 1)(2\ 4)$ and $\tau = (5\ 4)(3\ 1\ 2)$ in S_5.	(5)
(vi)	Define equivalence relation .Give example for equivalence and non - equivalence relation with an example.	(5)

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