B.N.BANDODKAR COLLGE OF SCIENCE – THANE

F.Y.B.Sc. (A.T.K.T) EXAMINATION – Feb.2011

MATHEMATICS PAPER – I

Duration: 3 Hrs Max. Marks : 90

N.B. 1. All Questions are compulsory.

SECTION I

Q.1 (a) Prove that I x.y I = I x I.I y I for x, y ϵ IR. (3)

(b) Attempt any three of the following.

(i) Let f(x) = $2x^2 + 4x + 2$, x ϵ IR. Sketch the graph of f. (4)

(ii) Using ϵ-δ definition, prove that (4)

$$\lim_{x \to 1} 2x+1=3$$

(iii) Discuss the continuity of (4)

$$F(x) = \frac{x^2-9}{x-3} \quad x \in [0, 3].$$

(iv) Prove that the equation $x^3 - 15x + 1 = 0$ has a root in [-4, 4]. (4)

(v) Let f(x) = x^2-1 if x < 2 (4)

= x+1 if x > 2

= 3 if x = 2

Determine whether f is continuous.

Q.2 (a) Find n^{th} derivative of f(x) = log (2x+3). (3)

(b) Attempt any three of the following.

(i) Let f : IR->IR be given by f(x)= sin x. Find the derivative of f using definition. (4)

(ii) Use Leibnitz rule to find n^{th} derivative of f(x) = x^3e^{4x}. (4)

(iii) Prove that every differentiable function is continuous. (4)

(iv) If f and g are differentiable then prove that f.g is differentiable. (4)

(v) State and prove Leibnitz rule. (4)

Q.3 (a) Prove that the function f(x) = $x^3 + 9x^2 + 30x + 7$ is increasing. (3)

(b) Attempt any three of the following.

(i) State and prove Lagrange's Mean Value Theorem. (4)

(ii) Find Taylor's expansion of f(x) = sin (x). (4)

(iii) Find Local Maxima/Minima of f(x) = $x^3 + 3x^2 - 24x$. (4)

(iv) Verify Roll's theorem for f(x) = x^2-1, x ϵ [-3, 3]. (4)

(v) Find an asymptote to the curve y = $\frac{x+1}{x+3}$. (4)

SECTION II

Q.4 (a) Find equation of a plane passing through (5,2,-1) having normal vector (1,-2, 1). (3)

(b) Attempt any three of the following.

(i) Find the projection of the vector (1, 1, 1) onto (3, -2, -1). (4)

(ii) Find equation of a line passing through the points (3, 2, 1) and (5, 4, 1). (4)

(iii) Express the equation r = sin ɵ + cos Θ in cartesian equation. (4)

(iv) Convert the rectangular coordinate (1, 2, 1) into spherical coordinate. (4)

(v) Define Orthogonal vectors . Prove that the vectors (2, 3,-1) and (1,-2,-4) are orthogonal. (4)

Q.5 (a) Using Polar coordinates prove that the following limit doesn't exist. (3)

$$\lim_{(x,y)\to(0,0)} \frac{4x^2}{7xy}$$

(b) Attempt any three of the following.

(i) Using definition prove that (4)

$$\lim_{(x,y)\to(0,0)} 2x + 3y + 1 = 1 .$$

(ii) (4)

Let $f(x, y) = \frac{2xy}{x^2+y^2}$ $(x,y) \neq (0, 0)$

$= 0$ $(x, y) = (0, 0)$

Find two different limits along two different path.

(iii) (4)

Let $f(x, y) = \frac{x+y}{x-y}$ $(x,y) \neq (0, 0)$

$= 0$ $(x, y) = (0, 0)$.

Determine whether f is continuous at (0,0).

(iv) Define continuity of a function f : IR^2 ->IR^2. (4)

(v) Let f : IR^2 -> IR^3 given by f(x, y) = (x + y, x – y, 2x + 3y). Prove that f is continuous at (1,2). (4)

Q.6 (a) Find all second order partial derivative of $f(x, y) = x^4 + 5x^3y + 4y^2 - 7y^3$. (3)

(b) Attempt any three of the following.

(i) Find linearization of $f(x, y) = x^3 – xy + y^2$ at (1, 1). (4)

(ii) Find directional derivative of f(x, y) = 2x + 3y at (1, 2) in the direction of(3, 4). (4)

(iii) Locate and classify maxima/minima of $f(x, y) = x^3 + 2y^3 - 3x^2 - 24y + 16$. (4)

(iv) Using Lagrange's Multiplier method , (4)

Maximize f(x, y) = x + y

Subject to $x^2 + y^2 = 1$.

(v) Use chain rule to find $\frac{dz}{dt}$, where $z = x^2 - xy - y^2$, $x = 1 - t^2$, $y = 3 + 2t$. (4)