A) Questions on Estimation

1 The following table shows the data about the number of seeds germinating out of 10 on damp filter paper which has Poisson distribution. Determine Estimate of λ.

Number of seeds	0	1	2	3	4	5	6	7	8	9	10
frequency	5	17	30	11	9	4	2	0	1	0	1

2 The life time of electric bulb is exponentially distributed; the data is collected on24 bulbs Determine Estimate of λ.

5.34,	2.87	18.03	33.12	56.81	45.12	11.23	9.09	21.47	10.24	78.91	8.10
1.26	43.29	52.81	27.23	15.29	3.06	1.93	38.77	42.44	20.76	63.64	14.15

3 The lead time follows gamma distribution estimate the parameters based on the following data.

3.125	8.137	1.015	13.674	5.526	4.241	10.392	19.309	2.374
6.667	14.211	6.619	7.301	16.021	15.556	1.011	9.713	12.911

4 The breaking strengths of steel bar follows normal distribution based on the following sample estimate parameters μ andσ^2.

345.15	481.32	614.14	732.75	545.78	464.21	809.94	792.07	662.35
517.55	669.88	441.59	556.79	787.63	693.25	522.72	381.28	489.46

5 The % of distribution of income size is log normally distributed determine estimate parameters μ andσ^2.

345.15	481.32	614.14	732.75	545.78
517.55	669.88	441.59	556.79	662.35

B) Distributions

1) Explain probability distribution for binomial variate. Where it can be applied.?

2) State probability distribution for Poisson variate. Where it can be applied.?

3) Write probability distribution function for geometric variate. Where it can be applied.?

4) On an average 2% defectives items are produced . A lot of size 50 is selected if it contains more than 2 defectives then lot is rejected. Compute the probability that lot is rejected.

5) 40% of the printers are rejected at the inspection centre. Find the probability that i) first accepted printer is the third one inspected. ii) five printers are inspected in order to accept three. iii) out of seven printers inspected four are accepted.

6) The calls due to the failure of a computer occur in accordance with Poisson distribution with a mean of 2 per day. Find the probability that i) there are three calls for computer failure on the next day. ii) Two or more calls on the next day. iii) At least one calls.

7) Of the orders a shop receives 25% are welding jobs and 75% are machining jobs. What is the probability that i) 2 out of next five jobs will be machining jobs? ii) Next four jobs will be welding jobs?

8) Students arrival at the library follows Poisson with mean 20 per hr. Determine the probability that i) there are 50 arrivals in the next one hr. ii) there are no arrivals in the next one hr. iii) there are 75 arrivals in the next 2 hrs.

9) Mr. A receives four calls in a day (Poisson distribution) .what is the probability that on the next day , the number of calls received will exceed the average by more than one standard deviation.

10) Arrivals at a bank teller's cage are Poisson distributed at the rate of 1.2 per minute.

    a. What is the probability of zero arrivals in the next minute?

    b. What is the probability of zero arrivals in the next 2 minutes?

11) In the production of ball bearing bubbles or depressions occur, then this bearing is unfit for sale. It has been noted that on an average one in every 800 of the ball bearing has a defect. What is the probability that a random sample of 4000 will yield fewer than three defective ball bearings?

12) A random variable X has pmf $P(x)=1/(n+1)$ over the range$\{0,1,2\ldots\ldots\ldots\ldots n\}$. Find mean and variance of this distribution.

13) An industrial chemical that will retard the spread of fire in paint has been developed. The local sales representative has determined, from past experience that 48% of the sales calls will result in an order.

a) What is the probability that the first order will come on the fourth sales call of the day?

b) What is the probability that the third order will come on the sixth sales call of the day?

c) If eight sales calls are made in a day, what is the probability of receiving exactly six orders?

d) If four sales calls are made before lunch, what is the probability that two or less results in an order?

14) Player A is currently winning 0.55 of his games. There are 5 games in the next two weeks. What is the probability that he will win more games than he will lose?

15) If r.v.x is geometrically distributed over 1.2…. with P (2) = 3P(3) Find P(x = odd)

16) A boy is throwing stones at a target, if the probability of hitting the target is 2/5. i)What is the probability that target it hit on the 5^{th} attempt? ii) find the probability that six trials were required to hit target thrice.

17) The demand (in Kg)for cakes is uniformly distributed over the range (1000, 2500) . Find the probability that on a given day demand is i) less than or equal to 1500 kg. ii) demand is between (1200, 2000) iii) demand is more than 2000 kg.

18) Demand for electricity at Gipgip Pig Farm for the merry merry month of May has a triangular distribution with a = 100 kwh and c = 1800 kwh. The median kwh is 1425. Determine the modal value of kwh for the month.

19) The current reading on Sag Reva's gas mileage indicator is an average of 25.3 miles per gallon. Assume that gas mileage on Sag's car follows a triangular distribution with a minimum value of zero and a maximum value of 50 miles per gallon. What is the value of the median?

20) The time intervals between dial-up connections to an Internet service provider are exponentially distributed with a mean of 15 seconds. Find the probability that the third dial-up connection occurs after 30 seconds have elapsed.

21) Suppose that the life of an industrial lamp, in thousands of hours, is exponentially distributed with failure rate λ = 1/3 (one failure every 3000 hours, on the average),find the probability that i) it will last longer than its mean life. ii) lamp will last between 2000 and 3000 hours iii) lamp will last for another 1000 hours, given that it is operating after 2500 hours.

22) For an exponentially distributed random variable X, find the value of λ that satisfies the following relationship:

$P\ (X \leq 2) = 0.9\ P\ (X \leq 3)$

23) The time to service customers at a bank's teller counter is exponentially distributed with mean 50 secs. What is the probability that the two customers in front of an arriving customer will each take less than 30 secs to complete their transactions?

24) The TV sets on display at Schocker's Department Store are hooked up such that when one fails, a model exactly like the one that failed will switch on. Three such units are hooked up in this series arrangement. Their lives are independent of one another. Each TV has a life which is exponentially distributed with a mean of 10,000 hours. Find the probability that the combined life of the system is greater than 32,000 hours.

25) The time to pass through a queue to begin self-service at a cafeteria has been found to be N (15, 9). Find the probability that an arriving customer waits between 14 and 17 minutes.

26) Demand for an item follows $N(50\ ,\ 7^2)$Determine the probability demand exceeds i) 45 units, ii)55 units, iii)65 units.

27) The annual rainfall in Chennai is normally distributed with mean 129 cms and standard deviation 32 cms. Evaluate the probability i) of getting excess rain(140cms or above)in a given year. ii) of getting 80cms or below rain in a given year.

28) Let X be a normal variable with mean 10 and variance 4 .Find the values of a and b such that $P(a<X<b)=0.90$ and $|\mu-a|=|\mu-b|$

29) Three shafts are made and assembled in to a linkage. The length of each shaft in cms is distributed as follows

Shaft1 X_1--N(60, 0.09) , Shaft2 X_2--N(40, 0.05) , Shaft3 X_3--N(50, 0.11)

i) Write down the distribution of the length of the linkage

ii)Determine the prob that length of the linkage will be longer than 150.2cms.

iii) The tolerance limits for assembly are(149.83, 150.21) , what proportion of the assembly are within the limits?

30) Given the following distribution evaluate $P(6<X<8)$

i) Normal(10,4) ii)Uniform (4,16) iii) exponential with parameter 0.1 iv) triangular (4,10,16)

31) State and prove Memory- less property of geometric distribution.

i) $P(X > s + t\ /\ X>t) = P(X>s)$ ii) $P(X = s + t\ /\ X>t) = P(X=s)$.

C)Inverse transformation technique

1. What is inverse transformation technique? Illustrate how it is used to generate random observations from exponential distribution.
2. For rectangular distribution ,Given a=2 and b= 4 generate six random observations using inverse transformation technique , use random numbers, 0.3, 0.48, 0.36, 0.01, 0.54, 0.34
3. Generation of five Exponential Variates X_i with Mean 1 using inverse transformation technique, use random numbers 0.13, 0.04, 0.65, 0.79, 0.76
4. Generation of six Exponential Variates X_i with λ=2, using inverse transformation technique use random numbers 0.30, 0.48, 0.36, 0.09, 0.54, 0.34.
5. Explain how inverse transformation technique is used to generate random observations from triangular distribution.
6.
7. Generate four random observations from triangular distribution over(0,1,2) using inverse transformation technique, use random numbers 0.15, 0.07, 0.65, 0.89,
8. **S**ervice time of a bank teller is found to follow normal distribution with mean 5 and s.d. 1. Generate five service times using inverse transformation technique, use random numbers 0.13, 0.04, 0.65, 0.79, 0.76
9. Consider the discrete distribution with pmf given by

$$p(x) = \frac{2x}{k(k+1)}, x = 1,2,..., k$$

Explain the procedure to generate random observation from this distribution.

10. Generate three values from a geometric distribution on the range $\{X \geq 1\}$ with mean 2. using inverse transformation technique .Such a geometric distribution has pmf $p(x)=p(1-p)^{x-1}$ (x = 1, 2,….)with mean $1/p = 2$, Using random numbers : $R_1 = 0.932$, $R_2 = 0.105$, and $R_3 = 0.687$.

11. Develop a random-variate generator for a random variable X with the pdf

$$f(x) = \begin{cases} e^{2x}, -\infty < x \leq 0 \\ e^{-2x}, 0 < x < \infty \end{cases}$$

12. Develop a generation scheme for the triangular distribution with pdf

$$f(x)=\begin{cases}\frac{1}{2}(x-2), 2\le x\le 3\\ \frac{1}{2}(2-\frac{x}{3}), 3< x\le 6\\ 0, otherwise\end{cases}$$

Generate 10 values of the random variate, compute the sample mean, and compare it to true mean of the distribution.

13. Given the following cdf for a continuous variable with range -3 to 4, develop a generator for the variable.

$$F(x)=\begin{cases}0, x\le -3\\ \frac{1}{2}+\frac{x}{6}, -3< x\le 0\\ \frac{1}{2}+\frac{x^2}{32}, 0< x\le 4\\ 1, x>4\end{cases}$$

14. Given the pdf $f(x) = x^2/9$ on $0 \le x \le 3$, develop a generator for this distribution.

15. Develop a generation scheme for the distribution with pdf f(x)

$f(x)=1/2 \quad 0<x<2$

$=1/24 \quad 2<x<10$

Generate five random observations, use random numbers 0.13, 0.04, 0.65, 0.79, 0.76

D) Examples on testing of RNs for independence.

1) The sequence of numbers 0.54,,0.73, 0.98 ,0.11 and 0.68 has been generated. Use *Kolmogorov-Smirnov test to determine whether the hypothesis that the numbers are uniformly distributed over (0,1) can be rejected. (*$D_{0.05}$=0.565)

2) Using chi square test with α=0.05 test whether the data shown below are uniformly distributed. Test is run for 10intervals of equal length.
0.34, 0.90, 0.89, 0.44, 0.46, 0.67, 0.83, 0.76, 0.70, 0.22,
0.96, 0.99, 0.17, 0.26, 0.40, 0.11, 0.78, 0.18, 0.39, 0.24
0.64, 0.72, 0.51, 0.46, 0.05, 0.66, 0.10, 0.02, 0.52, 0.18,
0.43, 0.37, 0.71, 0.19, 0.22, 0.99, 0.02, 0.31, 0.82, 0.67
0.46, 0.55, 0.08, 0.16, 0.28, 0.53, 0.49, 0.81, 0.64, 0.75

3) Based on runs up and runs down, determine whether the following sequence of 40 numbers is such that the hypothesis of independence can be rejected where α = 0.05.

0.41	0.68	0.89	0.94	0.74	0.91	0.55	0.62	0.36	0.27
0.19	0.72	0.75	0.08	0.54	0.02	0.01	0.36	0.16	0.28
0.18	0.01	0.95	0.69	0.18	0.47	0.23	0.32	0.82	0.53
0.31	0.42	0.73	0.04	0.83	0.45	0.13	0.57	0.63	0.29

4) Consider the following sequence of 40 numbers.
0.90, 0.89, 0.44, 0.21, 0.67, 0.17, 0.46, 0.83, 0.79, 0.40, 0.94, 0.22, 0.66, 0.42, 0.99, 0.67, 0.41, 0.73, 0.02, 0.72, 0.43, 0.47, 0.17, 0.56, 0.45, 0.78, 0.56, 0.30, 0.71, 0.19, 0.93, 0.37, 0.42, 0.96, 0.73, 0.47, 0.60, 0.29, 0.78, 0.26
Based on the runs ups and downs, determine whether the hypothesis of independence (random) can be rejected

5) Determine there is an excessive number of a run above and below the means (test the hypothesis that the numbers are independent)for the sequence of numbers given

0.41	0.68	0.89	0.94	0.74	0.91	0.55	0.62	0.36	0.27
0.19	0.72	0.75	0.08	0.54	0.02	0.01	0.36	0.16	0.28
0.18	0.01	0.95	0.69	0.18	0.47	0.23	0.32	0.82	0.53
0.31	0.42	0.73	0.04	0.83	0.45	0.13	0.57	0.63	0.29

6) Given the following sequence of numbers, can the hypothesis that the numbers are independent be rejected on the basis of the length of runs up and down at α=0.05?

0.30	0.48	0.36	0.01	0.54	0.34	0.96	0.06	0.61	0.85
0.48	0.86	0.14	0.86	0.89	0.37	0.49	0.60	0.04	0.83
0.42	0.83	0.37	0.21	0.90	0.89	0.91	0.79	0.57	0.99
0.95	0.27	0.41	0.81	0.96	0.31	0.09	0.06	0.23	0.77
0.73	0.47	0.13	0.55	0.11	0.75	0.36	0.25	0.23	0.72
0.60	0.84	0.70	0.30	0.26	0.38	0.05	0.19	0.73	0.44

7) Given the following sequence of numbers, can the hypothesis that the numbers are independent be rejected on the basis of the length of runs above and below the mean at $\alpha=0.05$?

0.30	0.48	0.36	0.01	0.54	0.34	0.96	0.06	0.61	0.85
0.48	0.86	0.14	0.86	0.89	0.37	0.49	0.60	0.04	0.83
0.42	0.83	0.37	0.21	0.90	0.89	0.91	0.79	0.57	0.99
0.95	0.27	0.41	0.81	0.96	0.31	0.09	0.06	0.23	0.77
0.73	0.47	0.13	0.55	0.11	0.75	0.36	0.25	0.23	0.72
0.60	0.84	0.70	0.30	0.26	0.38	0.05	0.19	0.73	0.44

8) Test whether the 3rd, 8th, 13th, and so on, numbers in the sequence at the beginning of this section are auto correlated.
0.12, 0.01, 0.23, 0.28, 0.89, 0.31, 0.64, 0.28, 0.83, 0.93, 0.99, 0.15, 0.33, 0.35, 0.91, 0.41, 0.60, 0.27, 0.75, 0.88, 0.68, 0.49, 0.05, 0.43, 0.95, 0.58, 0.19, 0.36, 0.69, 0.87