B. N. Bandodkar College of Science, Thane.
Junior College
S.Y.J.C. 1st Term Examination, October 2014
Subject – Mathematics

Day: Saturday **Time: 11.00 am to 01.00 pm**
Date: 11/10/2014 **Max. Marks: 50**

Note: 1) All questions are compulsory.
2) Figure to the right indicates full marks.

Q.1 A. Select the most appropriate answer from the given alternatives for each sub question: **06**

i) The negation of the statement $p \leftrightarrow q$ is
a) $(p \vee q) \wedge (\sim p \wedge \sim q)$ b) $(p \wedge q) \vee (\sim p \wedge \sim q)$
c) $(p \wedge \sim q) \vee (q \wedge \sim p)$ d) $(p \vee \sim q) \wedge (q \vee \sim p)$

ii) The principal solution of $\cos x = \frac{-1}{2}$ is
a) $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$ b) $x = \frac{\pi}{3}$ and $x = \frac{4\pi}{3}$
c) $x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$ d) $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

iii) If A(5,1,p), B(1,q,p) and C (1,-2,3) are vertices of triangle and G(r, $\frac{-4}{3}$, $\frac{1}{3}$) is its centroid, then values of p, q and r are,
a) -1, -3, $\frac{7}{3}$ b) 1,-3, $\frac{7}{3}$ c) 1, 3, $\frac{7}{3}$ d) 2, 4, $\frac{7}{3}$

B. Attempt *any Three* of the following: **06**

i) Construct switching circuit of the statement
$$[(p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$$

ii) Find the polar co-ordinates of the points whose Cartesian co-ordinates are $(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$

iii) Find the value of p, if the vectors $\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $2\hat{\imath} - 5\hat{\jmath} + p\hat{k}$ and $5\hat{\imath} - 9\hat{\jmath} + 4\hat{k}$ are coplanar.

iv) If a line makes angles α, β, γ with co-ordinate axes, prove that
1) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$ 2) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

v) Find the equation of line passing through the points (5, 4, 3) and having direction ratios -3, 4, 2

Q.2 A. Attempt *any Three* of the following: **09**

i) Find the inverse of $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ by using elementary row transformations.

ii) Using truth tables, prove that: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

iii) If $\bar{a} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\bar{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\bar{c} = 3\hat{\imath} - \hat{k}$ then, find the scalars m and n such that $\bar{c} = m\bar{a} + n\bar{b}$

iv) Find the angle between the lines whose direction cosines are given by the equation
$$6mn - 2nl + 5lm = 0$$
$$3l + m + 5n = 0$$

B. Attempt *any ONE* of the following: **04**

i) Solve the following equation by the method of inversion
$$x + y + z = -1$$
$$x - y + z = 2$$
$$x + y - z = 3$$

ii) In ΔABC, if a = 18, b = 24 and c = 30 then find,
a) $\sin\frac{A}{2}$ b) $\tan\frac{A}{2}$ c) A(ΔABC) d) sin A

Q.3 A. Select the most appropriate answer from the given alternatives for each sub-question: 06

i) If the function f is defined by $f(x) = \frac{(e^{3x}-1)\sin x^o}{x^2}$, for $x \neq 0$ is continuous at $x = 0$, then f(0) = _____

a) $\frac{\pi}{60}$ b) 3 c) $\frac{\pi}{20}$ d) -2

ii) If x = t log t, $y = t^t$ then $\frac{dy}{dx}$ = _____

a) e^x b) log x c) x d) x^2

iii) $\int \log x \, dx$ = _____

a) x [log x + 1] + c b) x [log x - 1] + c c) x [log x + x] + c d) x [log x - x] + c

B) Attempt *any three* of the following 06

i) Examine the continuity of the following functions

$$f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}} \text{ for } 0 < x \leq \frac{\pi}{2}$$
$$= \frac{\cos x}{\pi - 2x} \text{ for } \frac{\pi}{2} < x < \pi$$
} at $x = \frac{\pi}{2}$

ii) Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ with respect to 'x'.

iii) Find the approximate value of $\log_{10}(1016)$. Given $\log_{10} e = 0.4343$

iv) Evaluate $\int \frac{1}{\sqrt{3x^2-4x+2}} dx$

v) Determine 'K' such that the following function is p. m. f

$$P(x) = \begin{cases} K\binom{4}{x}, x = 0, 1, 2, 3, 4 ; K > 0 \\ 0 \text{ otherwise} \end{cases}$$

Q.4 A. Attempt *any Three* of the following: 09

i) If f(x) is continous on $[0, \pi]$, where

$$f(x) = x + a\sqrt{2}\sin x, \quad for\ 0 \leq x \leq \frac{\pi}{4}$$
$$= 2x\cot x + b, \quad for\ \frac{\pi}{4} < x \leq \frac{\pi}{2}$$
$$= a\cos 2x - b\sin x, \quad for\ \frac{\pi}{2} < x \leq \pi$$

find 'a' and 'b'.

ii) A rectanglar sheet of paper has the area 24 sq. meters. The margin at the top and bottom is 75 cm and sides 50 cm each. What are the dimensions of paper if the area of the printed space is maximum?

iii) Evaluate $\int sec^3 x \ dx$

iv) A random variable x has the following probability distribution:

x	0	1	2	3	4	5	6
p(x)	K	3K	5K	7K	9K	11K	13K

a) Find K, $p(x \geq 2)$, $p(0 < x < 4)$ b) Obtain the c. d. f of x

B. Attempt *any ONE* of the following: 04

i) If $y = [\log(x + \sqrt{x^2+1})]^2$ show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$

ii) Evaluate $\int \frac{3x-2}{(x+1)^2(x+3)} dx$

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