

B.N.BANDODKAR COLLEGE OF SCIENCE – THANE

S.Y.B.Sc First Term End Examination October – 2011

Subject – Mathematics

Paper I

Duration – 2 Hrs

Max. Marks 60

N.B. All questions are compulsory.

- Q.1 (a) Prove that the sequence $(\frac{1}{n})$ is a Cauchy sequence. (3)
- (b) **Attempt any three of the following.**
- (i) If $(x_n) \rightarrow p$ and $(y_n) \rightarrow q$ then prove that $(x_n + y_n) \rightarrow p + q$. (4)
- (ii) Find disjoint neighbourhood of 0.9 and 0.99. (4)
- (iii) Prove that the set $\{0\}$ is closed. (4)
- (iv) Write the definition of convergence of series. Prove that the series $\sum \frac{1}{n(n+1)}$ converges. (4)
- (v) Define open cover of a set. Find open cover for $(0, 1)$. (4)
- Q.2 (a) Prove that the set $S = \{x \in \mathbb{R} / x < 1\}$ is an Open set. (3)
- (b) **Attempt any three of the following.**
- (i) Prove that 0 is a limit point of a set $\{\frac{1}{n} \mid n \in \mathbb{N}\}$ (4)
- (ii) If A and B are closed subset of \mathbb{R} then prove that $A \cap B$ is closed. (4)
- (iii) State and prove Archimedian property. (4)
- (iv) Prove that if a Supremum of a set exist then it is unique. (4)
- (v) If $a < b$ and $c < d$ then prove that $a + c < b + d$. (4)
- Q.3 (a) Prove that if a sequence converges then it converges to unique limit. (3)
- (b) **Attempt any three of the following.**
- (i) Using definition, prove that $(\frac{1}{2n+1}) \rightarrow 0$ (4)
- (ii) State and prove Sandwich theorem of limit. (4)
- (iii) If f is continuous at a and g is continuous at a then prove that $f + g$ is continuous at a. (4)
- (iv) Prove that the sequence $(a^n) \rightarrow 0, 0 < a < 1$. (4)
- (v) Find limit superior and limit inferior of the sequence $(\frac{1}{n})$. (4)
- Q.4 (a) Define and prove Cauchy Criterion for convergence of series. (3)
- (b) **Attempt any three of the following.**
- (i) State and prove Comparison test. (4)
- (ii) Find Fourier series of $f(x) = |x|, x \in [-\pi, \pi]$. (4)

P.T.O.

(iii) Find radius of convergence of $\sum \frac{1}{n(n+1)} x^n$. (4)

(iv) Prove that $\sum \frac{1}{\sqrt{n}}$ does not converge. (4)

(v) Discuss the convergence of the series $\sum \frac{2n+1}{n^4+3}$. (4)
