B. N. BANDODKAR COLLEGE OF SCIENCE, THANE - 400 601.

FIRST TERM EXAMINATION OCT. - 2010

S. Y. B. Sc.

TIME : 2 Hrs.

SUBJECT : MATHEMATICS - II **MARKS :** 60

N. B. : 1. All questions are compulsory.

Q.1 a) Let V be a real vector space. A non-empty subset Wof V is subspace of V iff α, b $\in \mathfrak{R}$ x, y $\in$ W $\Rightarrow \alpha x + \beta y \in$ W. [3]

b) Attempt ANY THREE of the following.

i) Define inner product space over $\mathfrak{R}$. Find the angle between (1, 1, -1) and (0, -1, -1) in $\mathfrak{R}^3$ with usual inner product. [4]

ii) Define elementary matrix
Describe the corresponding elementary row operation and write the inverse of [4]

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

iii) If S is a subset of a vector space V over $\mathfrak{R}$ then the set L (S) is a subspace of V and L (S) is the smallest subspace of V containing S. [4]

iv) Define rank of a matrix.

Is $A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 0 & 3 \\ 4 & 1 & 4 \end{bmatrix}$ invertible?

Also find rank of A. [4]

v) Explain Gram-Schmidt method of orthogonalisation. [4]

Q.2. a) Prove that an n x n matrix A is invertible if and only if A is row equivalent to identity matrix. [3]

b) Attempt ANY THREE of the following :

i) Solve the following equations by Gauss-elimination method.

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 + x_2 + 3x_3 = 13$$
$$3x_1 + 3x_2 + 4x_3 = 20$$
[4]

ii) Express A as a product of elementary matrices [4]

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 5 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii) Prove that a necessary and sufficient condition for the sum of two solutions or any scalar multiple of a solution to be the solution of the same system of linear equations [4]

$$\sum_{j=1}^{n} a_{ij}x_j = b_i \qquad 1 \leq i \leq m$$

is that $b_i = 0$ for $1 \leq i \leq m$

iv) Show that the following systems of linear equations have infinitely many solutions and give geometrical interpretation. [4]

$$3x + y + z = 0$$
$$x + y + z = 0$$

v) If $A \in M_{mxn}(\mathfrak{R})$, $B \in M_{nxp}(\mathfrak{R})$ then prove that $(AB)^t = B^tA^t$ [4]

Q.3 a) If V is finitely generated vector space over $\mathfrak{R}$. W is a subspace of V then prove that W is finitely generated and $\dim W \leq \dim V$. [3]

b) Attempt ANY THREE of the following.

i) Show that the set { (1, 2, 3), (2, 1, 3)} is linearly independant. Extend it to basis of $\mathfrak{R}^3$. [4]

ii) Is $W = [(a, b, c) \in \mathfrak{R} \mid a \leq b]$ a subspace of $\mathfrak{R}^3$? [4]

iii) Define maximal L. I. set. [4]
Let $x = \{x_1, x_2, ... x_n\}$ be a finite set of vectors in V then if X is a maximal linearly independant set in V then prove that X is a minimal set of generators of V.

iv) Prove that $\{e^t, e^{2t}\}$ is L.I on ([0,1]). [4]

v) Define basis and dimension of a vector space.
If $W = \{ (x_1, x_2) \in \Re^2 \mid x_1 + x_2 = 0\}$ find basis and dimension of W. [4]

Q.4 a) State and prove : Cauchy - Schwartz inequality. [3]

b) Attempt ANY THREE of the following :

i) Prove that a finite dimensional real vector space has an orthonormal basis. [4]

ii) Use Gram - Schmidt method to find an orthogonal basis of $\Re^3$ for $\{ (0, 1, 1,), (1, -1, 0), (2, 0, 1) \}$ [4]

iii) If V is a real inner product space and $x, y \in V$ then
I) $4<x, y> = || x + y ||^2 - || x - y ||^2$ and
II) $|| x + y ||^2 + ||x - y ||^2 = 2|| x ||^2 + 2 ||y||^2$ [4]

iv) Show that { 1, x} is orthogonal subset of c [-1, 1] w.r.t. to inner product $<f (x), g (x) > = \int_{-1}^{1} f (x) . g (x) . dx$. [4]

v) If V is a real inner product space and u is a unit vector in V. Then for any $\vartheta \in V$.
$|| \vartheta - P_u (\vartheta) || \leq || \vartheta - \alpha u || \ \forall \alpha \in \Re$. [4]

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