(2½ Hours) **[Total Marks : 60**

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **marks** for respective **subquestions**.

1. (a) State and prove Havel-Hakimi Theorem for degree sequence of a simple graph. (7)

   (b) Attempt any **two** questions:

   (i) For any graph G with at least 6 vertices, prove that either G or $\overline{G}$ contains a triangle. (4)

   (ii) Define self complementary graph. Does there exists a self complementary graph with 98 vertices? Justify your answer. (4)

   (iii) If $(A^n) = (a_{ij}^n)$ is the n^{th} power of the adjacency matrix A of a graph G with $V(G) = \{v_1, v_2, \cdots, v_p\}$ then prove that the number of triangles in G is $\frac{1}{6}$ trace of A^3. (4)

   (iv) Prove that if G is a non trivial tree then there exists at least 2 vertices which are pendant vertices. (4)

2. (a) State and prove Cayley's Formula for spanning trees. (7)

   (b) Attempt any **two** questions:

   (i) Show that the number of leaves in a binary tree with n vertices is $\frac{n+1}{2}$. (4)

   (ii) Show that a vertex v in a tree T is a cut vertex of T if and only if $\deg(v) > 1$. (4)

   (iii) Prove that the vertex connectivity of a graph is less than or equal to edge connectivity of the graph. (4)

   (iv) Characterize the trees that are complete bipartite graphs. (4)

3. (a) Show that the cube graph Q_n, with $n \geq 2$ is a Hamiltonian graph. (7)

   (b) Attempt any **two** questions:

   (i) Show that the Hamiltonian closure of a graph is well defined. (4)

   (ii) Let G be a simple graph with p vertices and q edges with $p \geq 3$. If $q \geq \frac{p^2-3p+6}{2}$ then prove that G is Hamiltonian. (4)

   (iii) Check whether the Wheel graph W_n and $K_{2n,n}$ are Hamiltonian graphs or not for $n \geq 4$. (4)

   (iv) If G is a Hamiltonian graph then for every non empty proper subset S of $V(G)$ prove that $\omega(G - S) \leq |S|$. (4)

4. Attempt any **three** questions:

   (a) Let G be a (p, q) graph. Show that the following are equivalent.
   (i) G is acyclic and $q = p - 1$. (ii) G is connected and $q = p - 1$. (5)

   (b) If G is a simple graph with p vertices, q edges and k components, then prove that $q \geq p - k$. (5)

   (c) If G is a graph on p vertices with $deg(u) + deg(v) \geq p - 1$ for every pair of non adjacent vertices u and v in G then show that G contains a Hamiltonian path. (5)

   (d) Let $\tau(G)$ denote the number of spanning trees of a graph G. If $e \in E(G)$ is not a loop then prove that $\tau(G) = \tau(G - e) + \tau(G.e)$. (5)