B.N.BANDODKAR COLLEGE OF SCIENCE,THANE(WEST)

T.Y.B.Sc. Preliminary Examination,Feb. 2011

Mathematics Paper III: Topology of Metric Spaces

Duration:3 hours Max.Marks:100

N.B. 1)All questions are compulsory.

2)Figures to the right indicate full marks.

Q.1 Attempt <u>any two</u> of the following questions.

a) Let (X,d) be a metric space.Show that a non empty subset G of X is open set in X if and only if it is the union of open balls in X. (10)

b) Show that any non empty subset of IR is connected with respect to the usual distance metric if and only if it is an interval. (10)

c) State and prove Bessel's inequality in C$[-\pi, \pi]$. (10)

Q.2 Attempt <u>any two</u> of the following questions.

a) Let (X,d) be a metric space and d_1 be a metric defined by

$$d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$$

prove that $(x_n) \rightarrow P$ in (X,d) if and only if $(x_n) \rightarrow P$ in (X,d_1). (10)

b) Let (X,d) be a metric space.Show that a subset D of X is dense in X if and only if $D \cap G \neq \phi$ for each open subset G of X. (10)

c) Let (X,d) be a complete metric space.Prove that a subspace (Y,d) of (X,d) is complete if and only if Y is closed subset of X. (10)

Q.3 Attempt <u>any two</u> of the following questions.

a) Let (X,d) and (Y,d') be metric spaces and f: X $\rightarrow$ Y be any function.Prove that f is continuous on X if and only if for every closed subset F of Y ,f^{-1}(F) is closed in X. (10)

b) Let (X,d) and (Y,d') be metric spaces. If f:X $\rightarrow$ Y is continuous on X then prove the following:

i) For each subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$

ii) For each subset B of Y, $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$ (10)

c)i)Using ε -δ definition prove that the function f:(**IR^2 ,d**) $\rightarrow$(**IR^2 ,d**) defined by f(x,y)=(y,x) is continuous,where d is Euclidean distance .

ii) Let (X,d) be metric spaces and f: X $\rightarrow$ X is continuous function then show that A= {x $\in$X/f(x)=x} is a closed set. (10)

Q.4 Attempt <u>any two</u> of the following questions.

a) State and prove Fejer's Theorem. (10)

b) i)Find the Fourier series of the function f(x)=$|\cos x|$, $-\pi \le x \le \pi$

ii)State and prove Weierstrass approximation theorem. (10)

c) If D_n(t) denotes Dirichlet's kernel and K_n(t) denotes Fejer's kernel then prove that $D_n(t) = \frac{\sin\left(n+\frac{1}{2}\right)t}{2\sin\left(\frac{t}{2}\right)}$, $K_n(t) = \frac{\sin^2\left(\frac{nt}{2}\right)}{2n\sin^2\left(\frac{t}{2}\right)}$,0<t<∞ . (10)

Q.5 Attempt <u>any two</u> of the following questions

a) Let K be a subset of IR^n .Prove that K is compact if and only if K is closed and bounded. (10)

b) (i)Prove that a closed subset of compact metric space is compact.

ii)If A and B are connected subsets of a metric space (X,d) and $A \cap B \neq \phi$, then prove that $A \cup B$ is connected. (10)

c) Let (X,d) be a metric space. If A is a connected subset of X and $A \subseteq B \subseteq \bar{A}$ then show that B is connected.Hence deduce that $\overline{A}$ is connected. (10)