B.N.Bandodkar College of Science,Thane

F.Y.B.Sc.(A.T.K.T.)Examination Feb 2011.

Mathematics Paper II

Duration:3 hours Max.Marks:90

Instructions to the candidates:

1.All questions are compulsory.

2.Figures to the right indicate full marks.

SECTION I

Q.1 *(a)* Prove that $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n$. *(3)*

(b) Attempt <u>any three</u> of the following questions

(i) State and prove Euclid's Lemma. (4)

(ii) Prove that the number of primes is infinite. (4)

(iii) Show that 497 and 3105 are coprime. (4)

(iv) Prove that,for any positive integer n, n(n+1) is divisible by 2. (4)

(v) If p/a and p/($a^2 + b^2$) , where p is a prime then prove that p/b. (4)

Q.2 (a) Show that f:IR $\rightarrow$ IR given by f(x)=2x + 5 , is a bijection. (3)

(b) Attempt <u>any three</u> of the following questions

(i) Define binary operation, with an example . (4)

(ii) State addition and multiplication principles of counting. (4)

(iii) Check whether '*' defined as a*b = a+2b, for all a,b $\in$ Z is commutative and associative. (4)

(iv) For f: IR $\rightarrow$ IR^+,given by f(x) = e^x , find the inverse. (4)

(v) If f:IR $\rightarrow$ IR given by f(x)=$\sin^2x$ and g:IR $\rightarrow$ IR given by g(x)=x+3.Find fog and gof. (4)

Q.3 (a) If ax $\equiv$ bx (modn) and if (n,x)=1 then prove that a $\equiv$ b(mod n). (3)

(b) Attempt <u>any three</u> of the following questions

(i) For any integer x ,show that $x^2 \equiv$ 0 or 1 (mod 4). (4)

(ii) Find the last integer of 7^{313}. (4)

(iii) Define Euler 's ø function.Find ϕ(30) (4)

(iv) Verify Wilson's theorem for p= 7. (4)

(v) Find the incongruent solutions of 4x $\equiv$ 12 (mod 20). (4)

SECTION II

Q.4 (a) Define equivalence relation with an example. (3)

(b) Attempt <u>any three</u> of the following questions

(i) Write down all the partitions of X={a,b,c,d,e} having k parts ,$1 \leq k \leq 4$. (4)

(ii) If A is a non empty subset of a finite set B ,then prove that IAI ≤ IBI. . (4)

(iii) For the recurrence relation $a_n = a_{n-1} + a_{n-2}$, $n \geq 3$,$a_1 = a_2 = 1$. Find a_4 and a_5. (4)

(iv) Solve the recurrence relation $a_n=5a_{n-1} - 6a_{n-2}$,$n \geq 3$ with $a_1=1$ and $a_2=5$. (4)

(v) Define countable and uncountable sets with an example each. (4)

Q.5 (a) How many different arrangements can be made by using all the letters of the word ALLISWELL. (3)

(b) Attempt <u>any three</u> of the following questions.

(i) Calculate the coefficient of $x^3y^3z^4$ in $(x+y+z)^{10}$. (4)

(ii) Find the number of integers between 1 and 250, that are not divisible by 2 and 3 (4)

(iii) Find the product (3 5 7)(2 3 1 6 5 4)(6 8 3 2 1) in S_8 . (4)

(iv) Find the inverse of (1 6 5)(2 3 4) in S_6. (4)

(v) Find the total number of derangements on 4- symbols. (4)

Q.6 (a) Find the quotient and remainder when x^4-3x^2+4x+8 is divided by x^2+2. (3)

(b) Attempt <u>any three</u> of the following questions

(i) Find the g.c.d. of x^8-1 and x^6-1. (4)

(ii) Decide whether g(x)= x+2 is a factor of $f(x) = 2x^3 +7x +5$. (4)

(iii) Find the multiplicity of each root of $f(x) = x^3-4x^2+5x-2$. (4)

(iv) Prove that a polynomial of degree n has atmost n roots. (4)

(v) Give the polar representation of z = 1 - i. (4)